Rejoinder to "No Evidence of Dark Energy Metamorphosis", astro-ph/0404468

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Abstract. In a recent paper (astro-ph/0311364) Alam *et al.* argued that the SNe data of Tonry *et al.* 2003 and Barris *et al.* 2003 appear to favour DE which evolves in time, provided no other priors are invoked. (The effect of invoking priors such as the age of the Universe, the values of H_0 and Ω_{0m} and CMB/LSS observations could modify this conclusion, as demonstrated in astro-ph/0403687 and other recent papers.) The approach adopted by Alam *et al.* to reconstruct the properties of DE was severely (and, as we shall show below – unfairly) criticized by Jönsson *et al.* in astro-ph/0404468. In this paper we re-examine the parametrisation used in astro-ph/0311364 and show that, contrary to the claims of Jönsson *et al.* , the results obtained from this reconstruction are robust and therefore representative of the true nature of dark energy.

1. Introduction

The Universe appears to be accelerating and the nature of dark energy (DE), which drives this acceleration, is a subject of much current debate among cosmologists. The cosmological constant is the simplest possibility, but evolving dark energy models have also been suggested and one of the goals of current cosmological studies is to differentiate between these different types of models. Type Ia supernovae treated as standardized candles provided the first indications that the expansion of the universe is accelerating [1, 2] and one expects that the nature of dark energy will be further revealed by the study of these objects in conjunction with other data sets (CMB, LSS etc.).

Recently there have been several data releases from the two supernova teams [3, 4, 5, 6], making the total number of supernovae almost double that known previously. Using these datasets, we reconstructed the dark energy density and equation of state in two papers [7, 8] (hereafter Papers I and II) in an attempt to understand the nature of dark energy. In these works, we found that the current supernova data appears to favour an evolving dark energy model with $w \lesssim -1$ at present and that at 2σ, the evolving model is at least as probable as the cosmological constant. This result soon found support in other works (see for instance [9, 10, 11, 12, 13, 14, 15, 16] etc.). Recently, in [17], doubts have been expressed as to the reliability of the fitting procedure

used in Papers I and II. Hence in this work we re-examine the reconstruction process used in these two papers to check whether the results obtained in Papers I & II were merely a consequence of the analysis or whether they do actually reflect a property of the universe.

2. Reconstructing Dark Energy from Supernova Data

The SNe observations measure the luminosity distance $d_L(z)$. From this, information about the cosmological parameters may be obtained through the Hubble parameter, which in a spatially flat universe is related to the luminosity distance quite simply by [18, 19, 20]

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1} . \tag{1}$$

The dark energy density is defined as :

$$\rho_{DE} = \rho_{0c}\left[\left(\frac{H}{H_0}\right)^2 - \Omega_{0m}(1+z)^3\right] , \tag{2}$$

where $\rho_{0c} = 3H_0^2/(8\pi G)$ is the present day critical density of an FRW universe, and Ω_{0m} is the present day matter density with respect to the critical density.

Information extracted from SNe observations regarding $d_L(z)$ therefore translates directly into knowledge of $H(z)$, the dark energy density, and, through [21]

$$q(x) = -\frac{\ddot{a}}{aH^2} \equiv \frac{H'}{H}x - 1 , \tag{3}$$

$$w(x) = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{(2x/3)\ H'/H - 1}{1 - (H_0/H)^2\Omega_{0m}\ x^3} \ ; \ x = 1 + z , \tag{4}$$

into knowledge about the deceleration parameter of the universe and the equation of state of dark energy.

One route to the reconstruction of DE lies in the construction of an ansatz for one of the three quantities: $H(z), d_L(z)$ or $w(z)$. The ansatz must of course be sufficiently versatile to accommodate a large class of DE models. In Papers I and II we have used a polynomial fit to dark energy density of the form :

$$h(x) = \frac{H(x)}{H_0} = \left[\Omega_{0m}x^3 + A_0 + A_1x + A_2x^2\right]^{\frac{1}{2}} , \ x = 1 + z , \tag{5}$$

where $A_0 + A_1 + A_2 = 1 - \Omega_{0m}$. Note that this ansatz should not be considered as a truncated Taylor series for $h^2(z)$. Rather, it is an interpolating fit for $h^2(z)$ having the right behaviour for small and large values of z. The number of terms in this fit is sufficient given the amount and accuracy of the present supernovae data. With more and better data in the future, more terms with intermediate (*e.g.,* , half-integer) powers of x may be added to it.

This is equivalent to the following ansatz for DE density (with respect to the critical density) :

$$\tilde{\rho}_{DE}(x) = \rho_{DE}/\rho_{0c} = A_0 + A_1x + A_2x^2 , \tag{6}$$
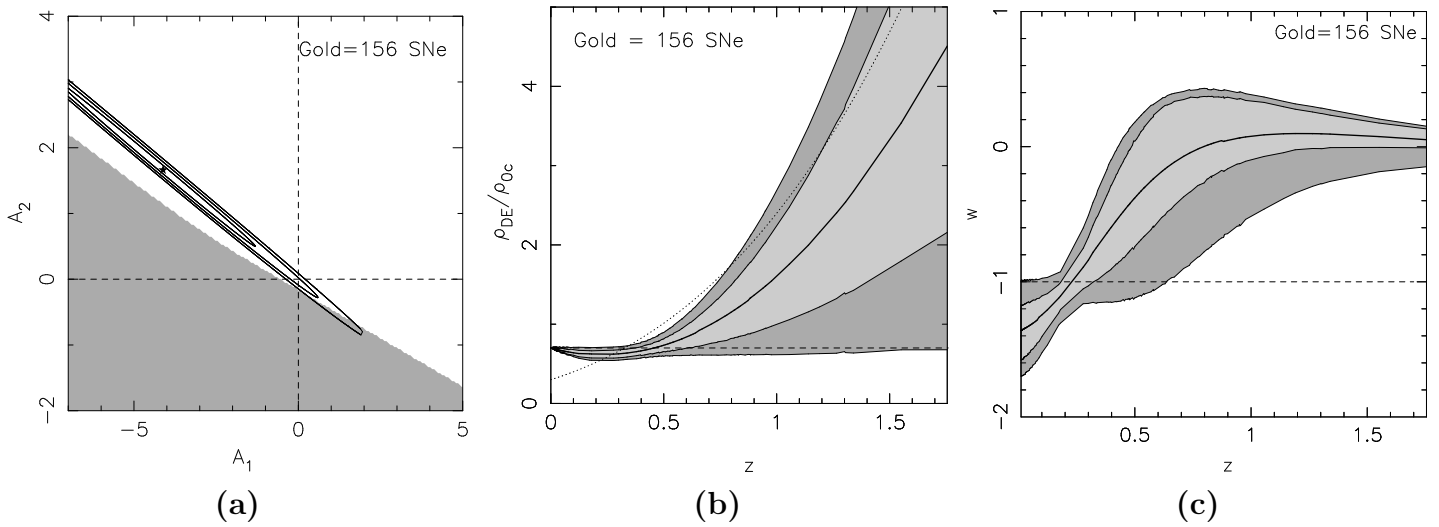
Figure 1. Results from analysis of the 156 SNe "Gold" data from [6], using ansatz (5). $\Omega_{0m} = 0.3$ and h is marginalised over. Panel (a) shows the $1\sigma, 2\sigma, 3\sigma$ confidence levels in the (A_1, A_2) parameter space. The star marks the best-fit and the ΛCDM point is at the intersection of the dashed lines. The shaded grey region has $\rho_{DE} \leq 0$ in the redshift range $0 < z \lesssim 2$. Panel (b) shows the variation of dark energy density $\rho_{DE}(z)/\rho_{0c}$ (where $\rho_{0c} = 3H_0^2/8\pi G$ is the present critical energy density) with redshift. Panel (c) shows the evolution of dark energy equation of state with redshift. In both the panels (b) and (c), the thick solid line represents the best-fit, the light grey contours represent the 1σ confidence level, and the dark grey contours represent the 2σ confidence levels. The dashed line in panels (b) and (c) represents ΛCDM, and the dotted line in panel (b) represents the matter density.

which is exact for the cosmological constant $w = -1$ ($A_1 = A_2 = 0$) and for DE models with $w = -2/3$ ($A_0 = A_2 = 0$) and $w = -1/3$ ($A_0 = A_1 = 0$).

The corresponding expression for the equation of state of DE is :

$$w(x) = -1 + \frac{A_1 x + 2A_2 x^2}{3(A_0 + A_1 x + A_2 x^2)} \ . \tag{7}$$

The likelihood for the parameters of the ansatz can be determined by minimising a χ^2-statistic:

$$\chi^2(H_0, \Omega_{0m}, p_j) = \sum_i \frac{[\mu_{\text{fit},i}(z_i; H_0, \Omega_{0m}, p_j) - \mu_{0,i}]^2}{\sigma_i^2} \ , \tag{8}$$

where $\mu_{0,i} = m_B - M = 5\log d_L + 25$ is the extinction corrected distance modulus for SNe at redshift z_i, σ_i is the uncertainty in the individual distance moduli (including the uncertainty in galaxy redshifts due to a peculiar velocity of 400 km/s), and p_j are the parameters of the ansatz used. We assume a flat universe for our analysis but make no further assumptions on the nature of dark energy. We also marginalise over the nuisance parameter H_0 by integrating the probability density $e^{-\chi^2/2}$ over all values of H_0.
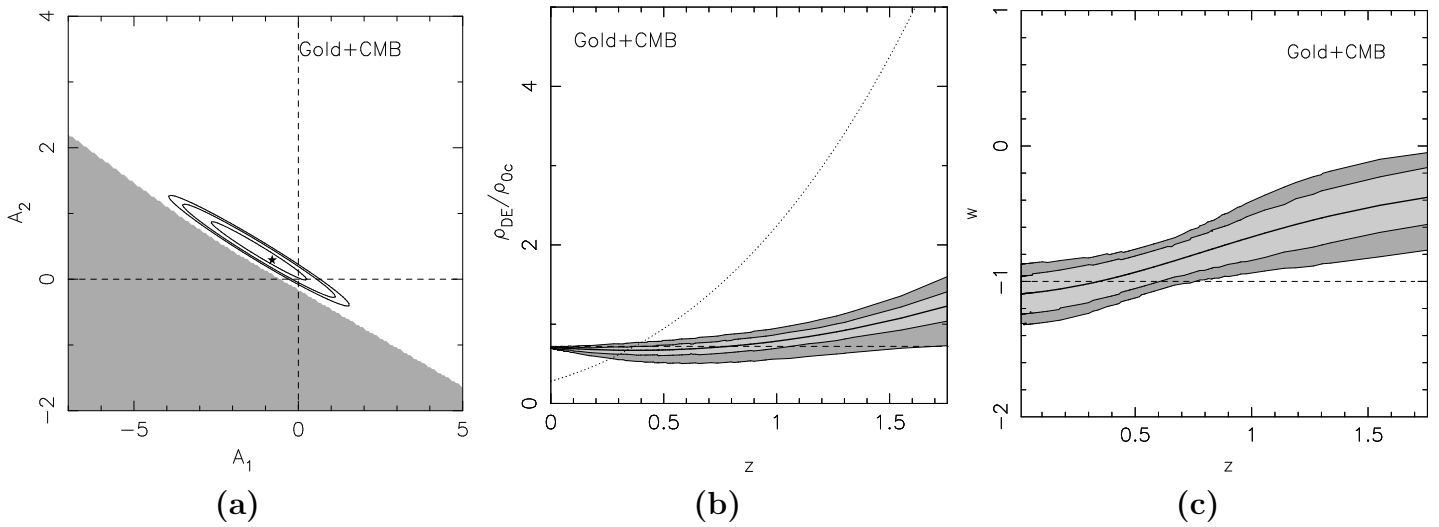
(a)   (b)   (c)

Figure 2. Results for analysis of SNe(Gold)+CMB data with ΛCDM-based WMAP priors of $\Omega_{0m} = 0.27 \pm 0.04$ and $h = 0.71 \pm 0.06$, using ansatz (5). Panel (a) shows the $1\sigma, 2\sigma, 3\sigma$ confidence levels in the (A_1, A_2) parameter space. The star marks the best-fit and the ΛCDM point is at the intersection of the dashed lines. The shaded grey region has $\rho_{DE} \leq 0$ in the redshift range $0 < z \lesssim 2$. Panel (b) shows the variation of dark energy density $\rho_{DE}(z)/\rho_{0c}$ (where $\rho_{0c} = 3H_0^2/8\pi G$ is the present critical energy density) with redshift. Panel (c) shows the evolution of dark energy equation of state with redshift. In both the panels (b) and (c), the thick solid line represents the best-fit, the light grey contours represent the 1σ confidence level, and the dark grey contours represent the 2σ confidence levels. The dashed line in panels (b) and (c) represents ΛCDM, and the dotted line in panel (b) represents the matter density.

Figure 1 shows the results obtained by using the ansatz (5) for the "Gold" dataset of 156 SNe published in [6]. We find that the cosmological constant is just within the 2σ confidence level in the $A_1 - A_2$ plane. The variation of dark energy density and equation of state of dark energy is also shown, and both show indications of evolution. We note that these results are stable to the addition of extra terms such as x^4 or x^{-1} terms to the ansatz. The region where the dark energy density becomes less than zero for the redshift range $0 < z \lesssim 2$ (the redshift range in which current supernova data is available) is shown as the shaded grey region in figure 1a. We see that a portion of the $3\sigma(99\%)$ confidence level lies in the region where dark energy density goes to zero and therefore w blows up.

Figure 2 shows the results of reconstruction when results from the WMAP experiment are incorporated and the priors $\Omega_{0m} = 0.27 \pm 0.04$ and $h = 0.71 \pm 0.06$ are assumed. In this case the time-evolution of DE is considerably weaker and more in agreement with a cosmological constant. (Note however that evolving DE is preferred over a cosmological constant by the combined SNe+WMAP data if $\Omega_{0m} \simeq 0.4, h \simeq 0.6$
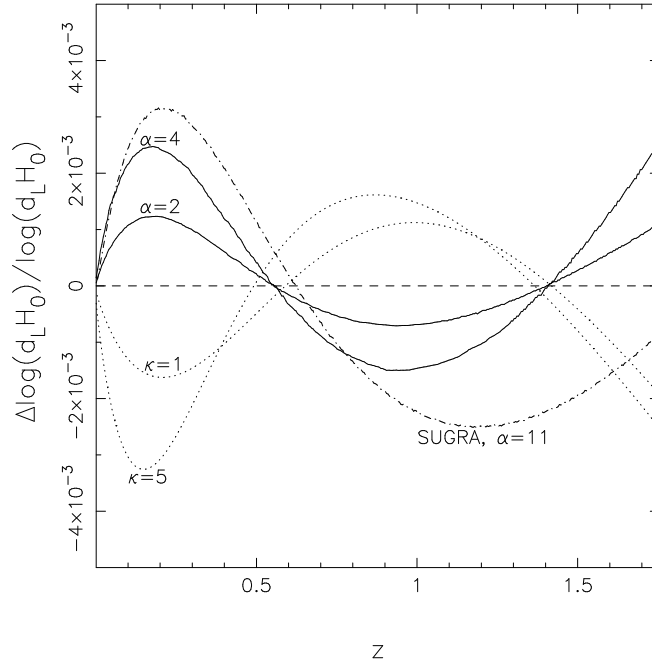
Figure 3. The fractional deviation $\Delta\log(d_L H_0)/\log(d_L H_0)$ between actual value and that calculated using the ansatz (5) over redshift for different models of dark energy with $\Omega_{0m} = 0.3$. The solid lines represent quintessence tracker models for potential $V = V_0/\phi^\alpha$, with $\alpha = 2$ and 4. The dotted lines show the deviation for Chaplygin Gas models with $\kappa = 1$ and 5 (where κ is the ratio between CDM and Chaplygin gas densities at the commencement of the matter dominated epoch). The dot-dashed line represents the SUGRA potential, $V = (M^{4+\alpha}/\phi^\alpha)\exp[\frac{1}{2}(\phi/M_{Pl})^2]$, with $M = 1.6 \times 10^{-8}M_{Pl}, \alpha = 11$. The dashed horizontal line represents zero deviation from model values, which is true for ΛCDM and $w = -1/3, w = -2/3$ quiessence models.

as shown in Paper II, [8].)

3. Examining the reconstruction exercise for possible bias

We now look at the reconstruction exercise in some detail to understand whether the results obtained in Papers I and II are biased due to inherent weaknesses in the ansatz used or whether these results can be trusted to reflect the true nature of dark energy.

3.1. Reconstruction of different dark energy models

In [17], the ansatz (5) was criticised for being biased and giving misleading information about the nature of dark energy. We show here that this is not the case and the ansatz is sufficiently robust to be a reliable probe of the nature of dark energy. In the figure 3, we show the *maximum deviation* between the actual value of the luminosity distance and that calculated using the ansatz (5) for simulations of different physically well motivated models of dark energy such as quintessence, Chaplygin gas, and the SUGRA potential.

From (5) it is clear that for $w = -1$, $w = -2/3$, and $w = -1/3$, this ansatz returns exact values. However, even for models for which it is not exact, the ansatz (5) still recovers the luminosity distance to within 0.5% accuracy in the redshift range relevant for SNe observations. These results reassure us that for a large class of models, the ansatz can be trusted to recover cosmological quantities to a high degree of accuracy. (Other tests of the ansatz (5) have been reported in [22, 23, 7].)

3.2. Diverging equation of state

In [17] the ansatz (5) has been criticized on the grounds that it allows the DE density to become negative during the course of cosmological evolution. We would like to emphasise that this is not necessarily a bad thing since there is no a-priori reason for the DE density to remain positive throughout its evolution and models do exist in which $\rho_{DE} \leq 0$ for a substantial duration of time – see for instance [24, 25, 26, 27, 28] etc. Indeed the purpose of the construction of an ansatz such as (5) is to endow it with sufficient flexibility so that it is able to reproduce the behaviour of a sufficiently large class of DE models.‡ In addition, a negative value for the dark energy density could also be an indication that the matter density has been over estimated, so one should not arbitrarily exclude the region of parameter space where $\rho_{DE} < 0$ as suggested by [17]. Hence we have not a priori put a constraint of positivity on the dark energy density in our analysis. However, one point to note is that the region where dark energy density vanishes is very close to the cosmological constant ($A_1 = A_2 = 0$) in parameter space, so there is a chance that if the best-fit to the data is near this point then the best-fit w or errors in it may blow up. This need not always happen, for instance in the figure 2, the best-fit is close to the cosmological constant, but the errors on w are well-behaved. The same is seen in figure 6 of Paper I. However, in case the best-fit equation of state or the errors in it do blow up, we naturally have to be careful about whether this truly indicates some exotic form of dark energy, or whether it is more consistent with the cosmological constant. It is also important to point out that the equation of state parameter need not always be the best characteristic of dark energy [23], and in such cases it is useful to characterize DE using other cosmological parameters (which do not exhibit this divergence), such as the deceleration parameter and the Statefinder pair [22, 23]. In this context it is worth noting that the pressure of dark energy : $P/\rho_{0c} = -\rho_{DE} + A_1 x/3 + 2A_2 x^2/3$ is well-behaved for this ansatz even when $\rho_{DE} < 0$, and this quantity can therefore be used to draw conclusions on the nature of dark energy.

3.3. Non-monotonic errors on the equation of state

The errors on the equation of state in figure 1(c) appear to be non-monotonic. This seems somewhat counter-intuitive. Low redshift behaviour of the equation of state affects the

‡ It is clear that no finite parameter ansatz will will ever be able to reproduce all possible behaviour of DE. For instance the ansatz (5) is unlikely to give good results if applied to DE which evolves extremely rapidly with time such as the rapidly oscillating models of [29].
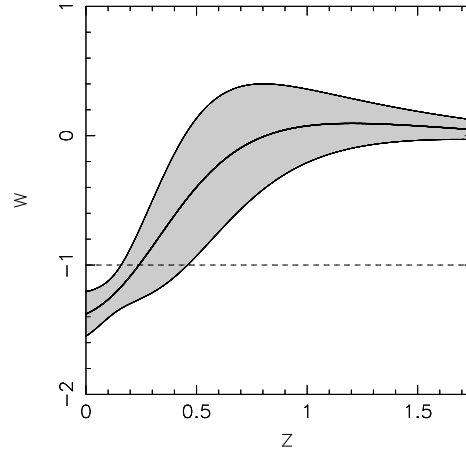
Figure 4. The variation of the equation of state of dark energy $w(z)$ over redshift for the ansatz (5). The thick solid line shows the best-fit and the light grey contour represents the 1σ confidence level around the best-fit. The dashed horizontal line denotes ΛCDM. $\Omega_{0\mathrm{m}} = 0.3$ is assumed. Errors are calculated by the Fisher matrix approach, using Eq 12.

luminosity distance at all higher redshifts, while high redshift behaviour affects fewer points. This leads to an expectation that high-z behaviour of the equation of state should be poorly constrained as opposed to the low-z behaviour. However, we saw in Papers I & II that the errors in $w(z)$ actually decrease with redshift. We investigate this matter using the Fisher matrix error bars.

In an analysis which uses an ansatz with n parameters p_i, the Fisher information matrix is defined to be

$$F_{ij} \equiv \left\langle \frac{\partial^2 \mathbf{L}}{\partial p_i \partial p_j} \right\rangle , \tag{9}$$

where $\mathbf{L} = -\log\mathcal{L}$, $\mathcal{L}$ being the likelihood . For an unbiased estimator, the errors on the parameters will follow the Cramér-Rao inequality : $\Delta p_i \geq 1/\sqrt{F_{ii}}$.

Since the likelihood function is approximately Gaussian near the maximum likelihood (ML) point, the covariance matrix for a maximum likelihood estimator is given by

$$(C^{-1})_{ij} \equiv \frac{\partial^2 \mathbf{L}}{\partial p_i \partial p_j} . \tag{10}$$

The Fisher information matrix is therefore simply the expectation value of the inverse of the covariance matrix at the ML-point.

Given the covariance matrix, the error on any cosmological quantity $Q(p_i)$ is given by :

$$\sigma_Q^2 = \sum_{i=1}^{n} \left(\frac{\partial Q}{\partial p_i} \right)^2 C_{ii} + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(\frac{\partial Q}{\partial p_i} \right) \left(\frac{\partial Q}{\partial p_j} \right) C_{ij} . \tag{11}$$

Thus the nature of the errors on a quantity will depend essentially on the manner in which it is related to the parameters of the system.

The errors on the equation of state for the ansatz (5) can now be calculated using equations (5) and (11), and has the somewhat complicated expression :

$$\sigma_w^2(x) = \frac{x^2[f_1^2 C_{11} + 2f_1 f_2 C_{12} + f_2^2 C_{22}]}{9[1 - \Omega_{0m} + A_1(x-1) + A_2(x^2-1)]^4} \; , \tag{12}$$

where

$$f_1 = 1 - \Omega_{0m} - A_2(x-1)^2 \; ,$$
$$f_2 = 2x(1 - \Omega_{0m}) + A_1(x-1)^2 \; ,$$

and A_1, A_2 are the best-fit values of the parameters.

From this expression it is difficult to predict whether the errors should increase with redshift or not. Indeed that would depend on the value and sign of the quantities C_{ii}. We calculate the errors around the best-fit for the 156 SNe "Gold" dataset of [6] using this procedure that the 1σ confidence levels thus calculated on the equation of state (shown in figure 4) are almost identical to those shown in figure 1(c). §

Thus the ansatz (5) need not show monotonically increasing errors in w with redshift and in this way may differ from other fitting functions, for instance approximations to the equation of state of dark energy as proposed in [30] or [31], which may show just the opposite tendency. This indicates that the nature of error bars are affected by the quantity being approximated. In the limit of infinite terms in the expansion of various quantities all the methods should produce identical result. The practical need for truncating these expansions make these approximations slightly different from each other. More specifically, we require setting of priors

$$f(z) = \sum_{i=0}^{\infty} a_n z^n \tag{13}$$
$$a_n \quad = 0; \; (n > N_p) \tag{14}$$

where $f(z)$ could be $H(z)$, $w(z)$ or any other physical quantity and N_p is the chosen number of parameters. The non-linear priors in the above equation make different finite expansions inequivalent. Since we do not know for certain if the underlying model for the accelerating expansion involves an energy component with negative pressure in a FRW setting we are forced to choose one of the alternatives for approximations. We hope that with increasingly high quality data the effect of such truncations will eventually disappear.

3.4. Stability of the dark energy equation of state

The dark energy equation of state for the ansatz (5) can have many different forms depending on the values of A_1, A_2. This ansatz gives us an exact representation of the cosmological constant ($A_1 = A_2 = 0$) and it is an unbiased estimator for the ΛCDM model. However, it is also true that the statistical noise inherently present in the data

§ One should note that the Fisher matrix approach works best for the cases where the likelihood is symmetric around the best-fit. For asymmetric likelihood curves, this method should not be used.

will never allow us to measure A_1, A_2 to be exactly equal to zero even in this case. Thus if the underlying model of the universe is the cosmological constant, then by fitting an ansatz which has more parameters than required, we are fitting a bit of noise as well. This fact is true for *any* ansatz and not just (5). The deviation of the best-fit from the cosmological constant in such a case is given by :

$$\delta w_\Lambda \simeq \frac{x}{3(1 - \Omega_{0\mathrm{m}})}\delta A_1 + \frac{2x^2}{3(1 - \Omega_{0\mathrm{m}})}\delta A_2 \ , \tag{15}$$

i.e. the w_Λ curve would increasingly depart from the cosmological constant with redshift. The same tendency would be seen if we consider a linear ansatz in the equation of state : $w = w_0 + w_1 z$. This would also diverge from the cosmological constant ($w_0 = -1, w_1 = 0$) at high redshifts for even a small non-zero δw_1, since $\delta w_\Lambda \simeq \delta w_0 + \delta w_1 z$. Such divergence is the inevitable price we pay for not knowing the "true" nature of dark energy and for using an ansatz to explore it. In practice one is free to choose either of two possibilities: (i) assume that the underlying model has certain characteristics (such as constant equation of state) and thus face the danger of losing information– the pitfalls of using this approach were highlighted in [32], or (ii) we may choose to make no such assumptions and consider models with a somewhat larger number of parameters. We have adopted (ii) in Papers I & II and believe that, given the redshift range and statistical noise of the data currently available, it is not unreasonable to assume that the errors for the ansatz (5) will not be large enough to cause the best-fit to diverge significantly from the underlying model irrespective of whether the model is that of the cosmological constant or some other form of dark energy. This is demonstrated in figure 7(a) for simulations of the cosmological constant model.

3.5. Correlation of the parameters A_1, A_2

In our analysis of the supernova data we find that for a fixed value of $\Omega_{0\mathrm{m}}$, the parameters A_1, A_2 which are favoured by the data are strongly correlated. For $\Omega_{0\mathrm{m}} = 0.3$, from the confidence levels of figure 1(a) we may conclude that approximately, $A_1 \simeq -2.4 A_2$ for points favoured by the data. Contrary to what is claimed in [17] this correlation is not something that has been put into the analysis by hand, rather it is a consequence of the observational data. Other fits, such as the Linder fit to the equation of state [30], have also shown a strong correlation between its parameters. We need to examine what effect, if any, this correlation has on the nature of dark energy. From expression (7) for the equation of state for dark energy, we see that $w = -1$ at a redshift of $z = -A_1/2A_2 - 1$. Therefore, for $\Omega_{0\mathrm{m}} = 0.3$, $w = -1$ at $z \simeq 0.2$. This implies that since most curves would cross $w = -1$ at approximately this redshift, there will be a "sweet-spot" at $z \simeq 0.2$ for the equation of state. This can be clearly seen in the figure 1(c). The presence of sweet-spots at different redshifts for different ansatz is something which has been studied earlier, for example in [33], and the community agrees that the presence or absence of a sweet-spot is simply a consequence of the ansatz used and does not signify any extra information about the underlying model. Does the fact that w crosses -1 at
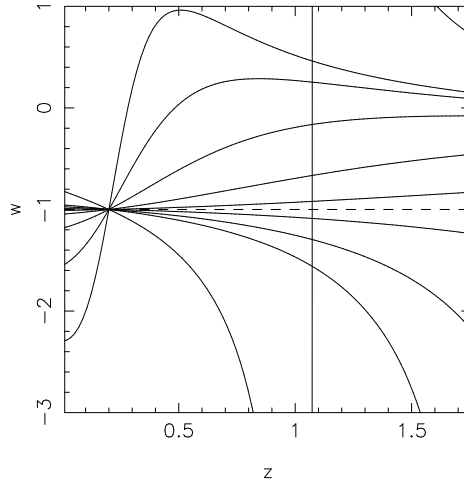
Figure 5. The variation of the equation of state with redshift for different values of the parameters A_1, A_2 satisfying $A_1 = -2.4A_2$. The thick dashed line denotes ΛCDM.

this redshift mean that the ansatz forces the equation of state to evolve in a particular direction? In figure 5, we plot the behaviour of w for different values of the parameters A_1, A_2 which follow the constraint $A_1 = -2.4A_2$. We see that the cosmological constant is allowed by this constraint, as are many other models. One can have extremely mild evolution of w, or more rapid evolution of w, one can have w which blows up, w which goes from a more negative (< -1) value to a less negative (> -1) value or vice versa. In short this constraint does not force the equation of state to evolve at a particular rate in a particular direction, but leaves a great deal of freedom. Also, it is important to note that this is not a constraint that has been imposed by hand in the analysis, it is simply an outcome of the analysis. There are many points in the A_1, A_2 parameter space for which the equation of state is always ≥ -1 and shows little evolution. The ansatz does not preclude these points, it is the data which chooses a particular evolving dark energy model over other possibilities. We therefore conclude that the correlation seen in the parameters A_1, A_2 does not signify any bias in the results and (contrary to the claims of [17]) the ansatz does not "force" evolution of the dark energy equation of state.

3.6. Confirmation from other fitting functions

In Paper I, we showed that the results obtained by the ansatz (5) can also be reproduced using other fitting functions, such as the fits to the equation of state used in [30] and [31]. For both these fits, the equation of state shows an evolution which is remarkably similar to that found using (5). It is interesting that our results have found support in the independent analysis carried out by other groups which also show that evolving DE provides a fit that is as good (or better) than that provided by a cosmological constant to the current supernova data [9, 10, 11, 12, 13]. Therefore there seems to be a consensus

among the different groups that, based on the SNe data alone, an evolving dark energy model is a plausible alternative to the cosmological constant. (As emphasised earlier in this paper and in Papers I and II, the properties of dark energy depend crucially upon the manner in which the SNe data is sampled and also upon which other sets of observations are used in conjunction with type Ia supernovae. For instance, the evolution of DE is least if one uses SNe+WMAP results together with the priors $\Omega_{0m} = 0.3, h = 0.7$; see also [34].)

4. Simulation of a ΛCDM model

To test the reliability of our ansatz, we attempt to reconstruct a fiducial cosmology from simulated data. We assume that the background model is a flat ΛCDM universe with $\Omega_{0m} = 0.3$, and generate 1000 datasets, each consisting of 156 SNe with the same redshift distribution and magnitude errors as those in the "Gold" dataset of [6]. We first analyse these datasets using the polynomial fit to dark energy density, Eq (5). The best-fit values of A_1, A_2 for these 1000 sets are shown in figure 6. The mean of this set of 1000 points is very close to the cosmological constant: $\bar{A}_1 = -0.01, \bar{A}_2 = 0.005$ ($A_1 = A_2 = 0$ for ΛCDM). The best-fit points consistent with the cosmological constant at 68% confidence level are indicated by filled circles, points consistent at 95% confidence level are shown by crosses, those consistent at 99% confidence level are shown by filled triangles, and those inconsistent at 99% confidence level are shown by the open circles. By determining confidence levels around the best-fit for each individual dataset we find that about 75% of the best-fit points are consistent with the cosmological constant at 1σ. We also note that some of the best-fit points are in the region where $\rho_{DE} \leq 0$. We now choose three of these 1000 datasets and calculate ρ_{DE} and w for the corresponding cosmology. The three best-fit points chosen are shown– the first (hereafter called dataset (a)) is consistent within 68% confidence level, the next (dataset (b)) is consistent at 99% confidence level and the last (dataset (c)) is inconsistent at 99% confidence level with the cosmological constant. The results are shown in figures 7 and 8. We find that for the dataset (a), ρ_{DE} and w are consistent with the cosmological constant, even though the errors in w blow up. For the datasets (b) and (c), the nature of dark energy density and dark energy equation of state appear to be very different from that of the cosmological constant. Such an example was used in figure 4 of [17] to make the claim that the ansatz (5) leads us to conclude that w is evolving even though the fiducial model is that of the cosmological constant.

Let us now see if this is really correct and whether our results were indeed a construct of the ansatz used.

- Let us consider the same fiducial model (ΛCDM) but an alternative ansatz to reconstruct dark energy. As our first example we consider a cosmological constant model but make no assumption about the flatness of the universe. The luminosity
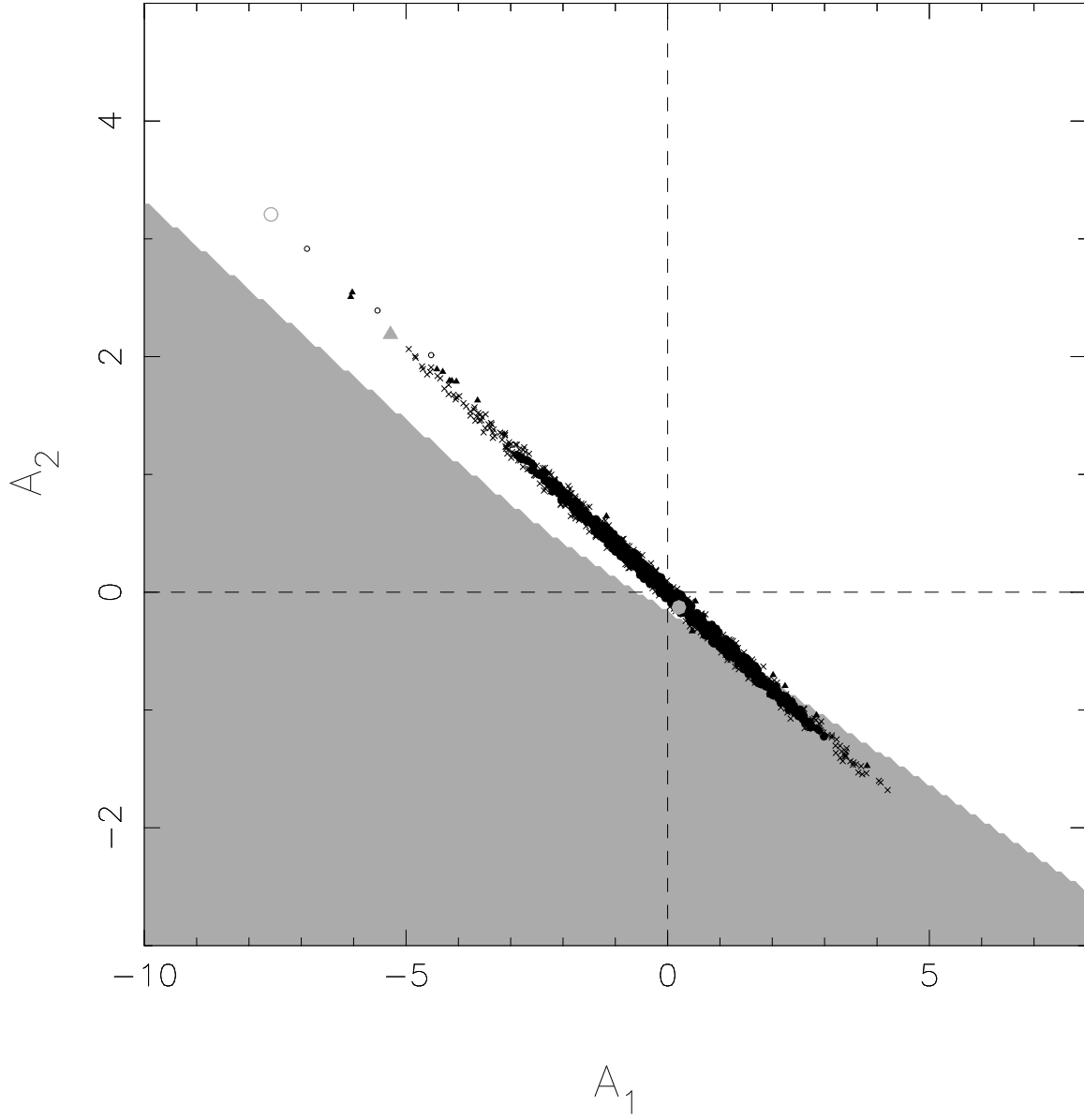
Figure 6. A scatter plot of the best-fit values A_1, A_2 of the polynomial fit to DE density, Eq (5), for 1000 simulated data sets in a flat universe with a cosmological constant. $\Omega_{0m} = 0.3$ is assumed. The filled circles represent simulated datasets consistent with ΛCDM at the 68% confidence level, the crosses represents those consistent at the 95% confidence level, the filled triangles are consistent at the 99% confidence level, while the open circles are inconsistent at the 99% confidence level. Three of these datasets used in further analysis are shown by (a) a grey circle, (b) a grey triangle, and (c) a grey open circle. The grey shaded region has $\rho_{DE} \leq 0$ in the redshift range $0 < z \lesssim 2$.
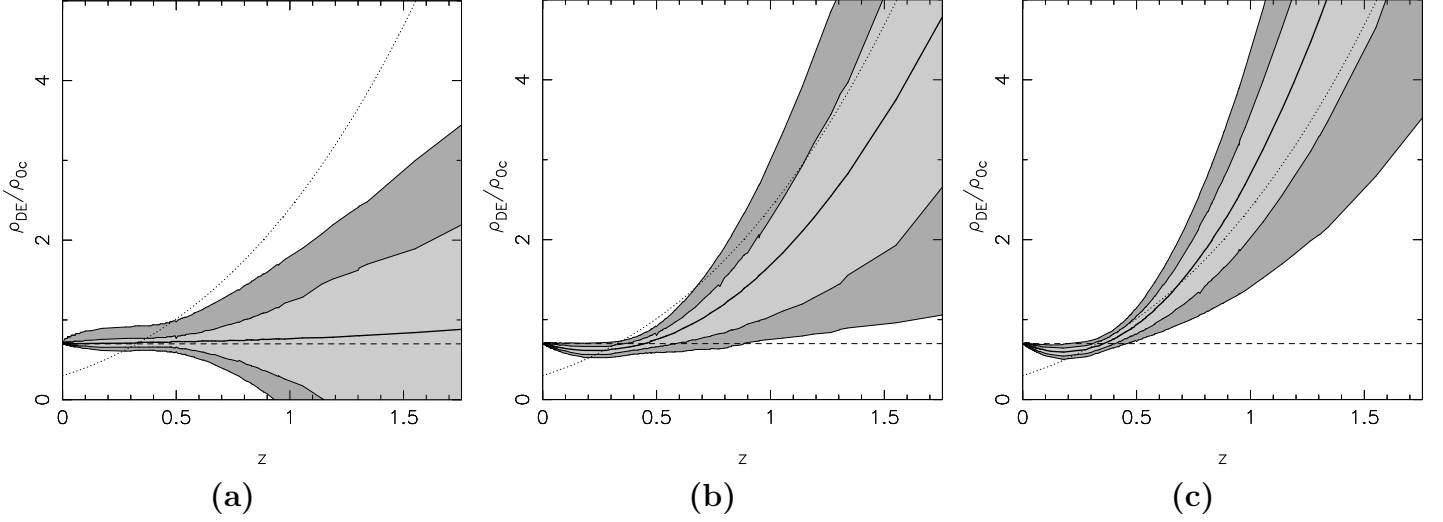
Figure 7. The variation of dark energy density $\rho_{DE}(z)/\rho_{0c}$ (where $\rho_{0c} = 3H_0^2/8\pi G$ is the present critical energy density) with redshift, using three sets of simulated data for a fiducial flat ΛCDM model. The polynomial fit to DE density, Eq (5) with $\Omega_{0m} = 0.3$ in a flat universe has been used. In each panel, the thick solid line shows the best-fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best-fit. The dotted line denotes matter density $\Omega_{0m}(1 + z)^3$, and the dashed horizontal line denotes ΛCDM.
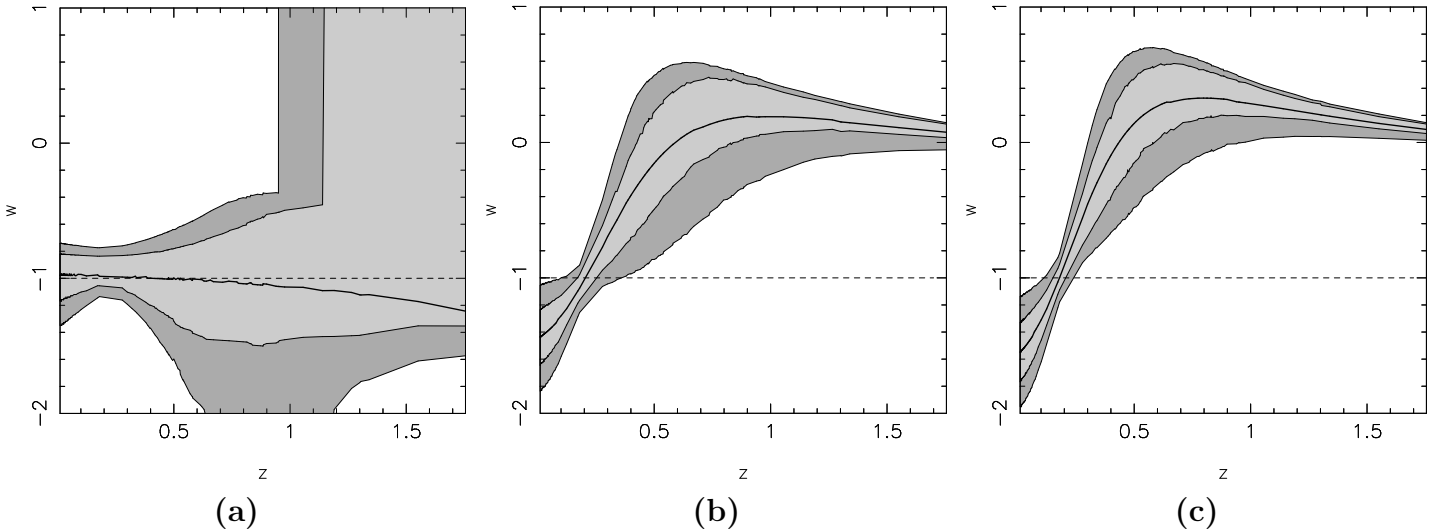

Figure 8. The variation of equation of state of dark energy $w(z)$ with redshift for $\Omega_{0m} = 0.3$ in a flat universe using three sets of simulated data for the polynomial fit to DE density, Eq (5). The background model is flat ΛCDM. In each panel, the thick solid line shows the best-fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best-fit. The dashed horizontal line denotes ΛCDM.
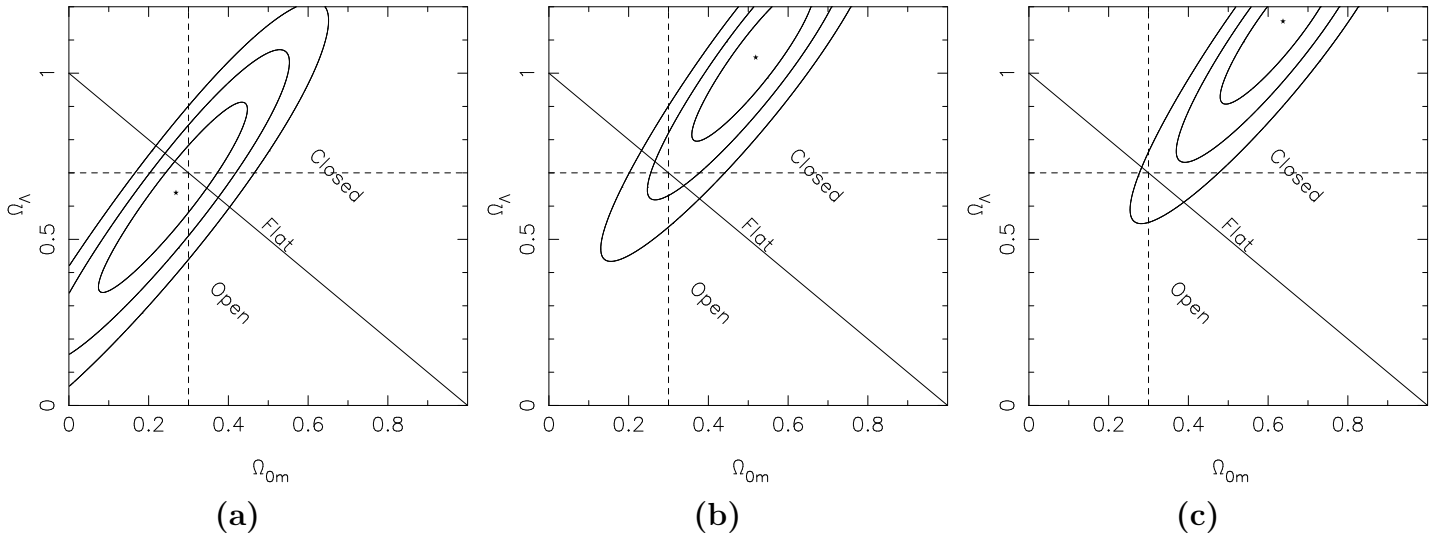
Figure 9. The $1\sigma, 2\sigma, 3\sigma$ confidence levels in $\Omega_{0\mathrm{m}}, \Omega_\Lambda$ for the three sets of simulated data (with a flat cosmological constant background model, $\Omega_{0\mathrm{m}} + \Omega_\Lambda = 1$) using a cosmological constant model, Eq (16) for the analysis. In each panel, a star marks the best-fit. The intersection of the dashed lines denotes ΛCDM. The solid line denotes the flat universe with $\Omega_{0\mathrm{m}} + \Omega_\Lambda = 1$. All universes above this line are closed and those below it are open.

distance in this case is given by :

$$d_L(z) = \frac{c(1+z)}{H_0\sqrt{|\Omega_\kappa|}} \mathcal{S}\left\{\sqrt{|\Omega_\kappa|}\int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_{0\mathrm{m}}z) - z(2+z)\Omega_\Lambda}}\right\} \quad (16)$$
$$(\Omega_\kappa = 1 - \Omega_{0\mathrm{m}} - \Omega_\Lambda; \mathcal{S} = \sin, 1, \sinh \text{ for } \Omega_\kappa <, =, > 0) .$$

The parameters of the system are $\Omega_{0\mathrm{m}}, \Omega_\Lambda$. The results are shown in the figure 9. We see that in case of dataset (a), the errors on $\Omega_{0\mathrm{m}}, \Omega_\Lambda$ are completely consistent with the cosmological constant in a flat universe, but for both the datasets (b) and (c) a closed model is the preferred model and the flat model (which is the true background model) is excluded at 1σ and 2σ respectively !

We may also consider different fitting functions for w in a flat universe.

- Consider the popular linear fitting function for the equation of state :

$$w(z) = w_0 + w_1 z \quad (17)$$
$$H^2(z) = H_0^2[\Omega_{0\mathrm{m}}(1+z)^3 + (1-\Omega_{0\mathrm{m}})(1+z)^{3(1+w_0-w_1)}\mathrm{e}^{3w_1z}] .$$

This is the ansatz used in [6] to parametrise dark energy. The resultant confidence levels in w_0, w_1 are shown in the figure 10. Once again, we see that only in case of the dataset (a) does the parametrisation come close to reconstructing the actual model. For the dataset (b), the cosmological constant is ruled out at 1σ and for the dataset (c) it is ruled out at 2σ.
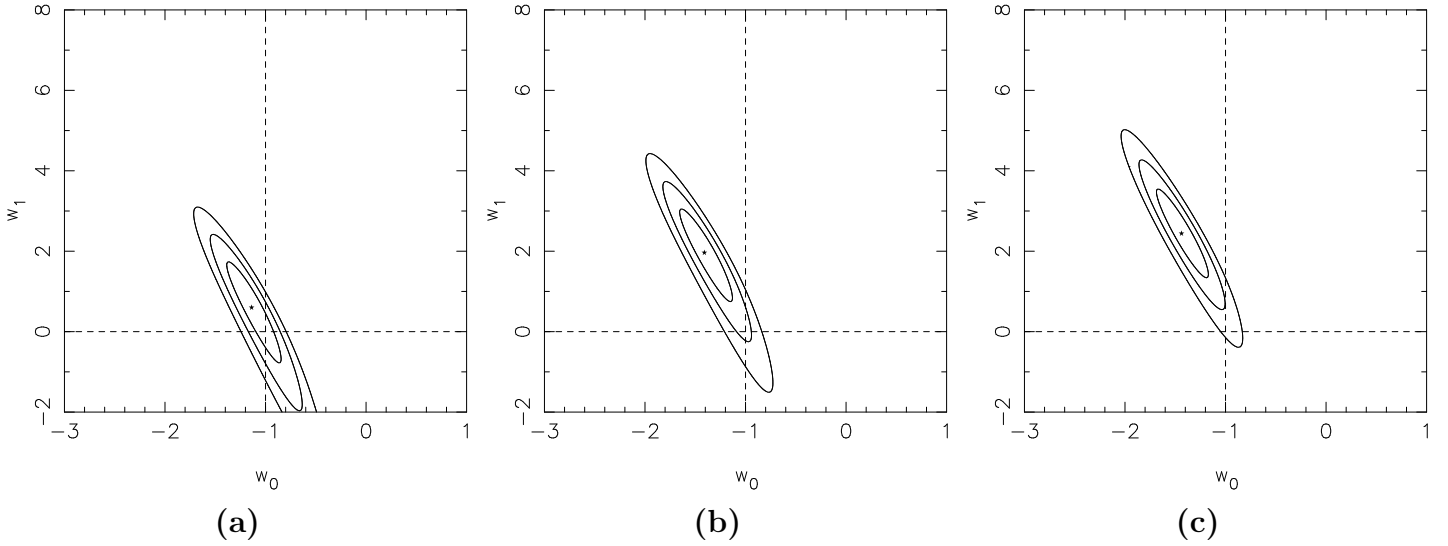
Figure 10.   The $1\sigma, 2\sigma, 3\sigma$ confidence levels in w_0, w_1 using three sets of simulated data for the linear fit to equation of state of DE, Eq (17). $\Omega_{0m} = 0.3$ and a flat universe are assumed. Background model is flat ΛCDM. In each panel, a star marks the best-fit. The intersection of the dashed lines denotes ΛCDM.
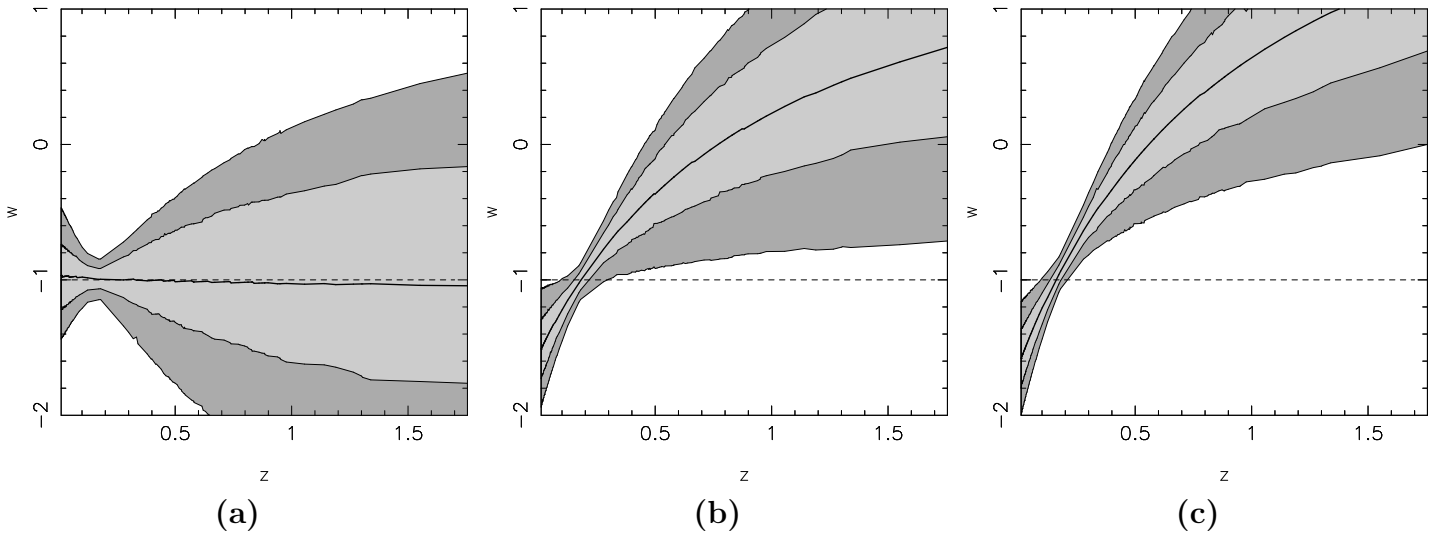

Figure 11. The variation of equation of state of dark energy $w(z)$ with redshift for $\Omega_{0m} = 0.3$ and a flat universe using three sets of simulated data for Linder's fit to the equation of state of DE, Eq (18). Background model is flat ΛCDM. In each panel, the thick solid line shows the best-fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best-fit. The dashed horizontal line denotes ΛCDM.

- Finally consider the parametrisation suggested in [30] :

$$w(z) \quad = w_0 + \frac{w_1 z}{1+z} \tag{18}$$

$$H^2(z) = H_0^2[\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})(1+z)^{3(1+w_0+w_1)}e^{-3w_1 z/(1+z)}] \ .$$

The evolution of the equation of state with redshift in this case for the three datasets is shown in the figure 11. We see that in the first panel (a), the evolution is very moderate and is entirely consistent with a cosmological constant. In the second and third panels however, we see significant evolution of the equation of state, which is consistent with what is seen in the panels (b) and (c) of figure 8, and inconsistent with a cosmological constant model.

The above examples clearly indicate that the different fits give results which are mutually consistent and that, contrary to the claims made in [17], the ansatz (5) does not "force" evolution of the equation of state of dark energy when the data is commensurate with the cosmological constant. If the datasets for which (5) shows evolution of DE are analysed using other fits, similar (misleading) conclusions about the behaviour of DE are reached. So these results are dependent not so much on the ansatz as on the dataset itself. Therefore it is in principle possible that even with a flat ΛCDM fiducial model having errors commensurate with current observations one can reconstruct dark energy which evolves, or a universe that is closed ! However, one must remember that out of the 1000 realisations of such a model, about 75% are broadly consistent with the cosmological constant model for the ansatz (5) and if these datasets are analysed then the correct model will be recovered within 1σ, as in case of the dataset (a). The datasets (b) and (c) which give rise to incorrect results are much less likely to occur. As the quality of data improves, the likelihood of obtaining datasets like (b) and (c) will further decrease. Given the current observational standards, it is reasonable to assume that the actual data which we have is consistent with the underlying model and results obtained from it are reliable. As more data becomes available, we will be able to discern the nature of the universe with greater accuracy.

5. Conclusion

We have shown that the results of Papers I and II are robust and that the conclusions drawn there are not due to any bias in the ansatz used. Contrary to the claims in [17] the ansatz (5) *does not* inherently "force" or "favour" evolving dark energy models over the cosmological constant. This ansatz can reproduce the behaviour of a large class of DE models fairly accurately and as such it can be used with confidence to predict the nature of the universe using currently available data. The conclusion that the current supernova data favours the evolving dark energy models over the cosmological constant at 1σ still holds and this result has found support in the work of other independent groups [9, 10, 11, 12, 13, 14, 15, 16] when no other data sets are used. Better quality data expected in the future from different cosmology experiments (SNe, CMB, LSS etc.) will allow us to draw firmer conclusions about the nature of dark energy.

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